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Discussion and Reply

Discussion on "Gravity anomalies of 2D bodies with variable density contrast" (Jianzhong Zhang, Benshan Zhong, Xixiang Zhou, and Yun Dai, 2001, GEOPHYSICS, 66, 809–813).

Discussion by Xiaobing Zhou[1]

In "Gravity anomalies of 2D bodies with variable density contrast," Zhang et al. derive a set of equations (9 and 10 in their paper) for a gravity anomaly at the origin of the coordinate system (x, z) for a polygon mass body when the density contrast is a polynomial function of x and z. For convenience of discussion, I reproduce and renumber their equations 9 and 10 as, respectively,

$$\Delta g(0,0) = -2G\sum_{i=0}^{N_x}\sum_{j=0}^{N_z}\frac{a_{ij}}{i+j+1}\sum_{k=1}^{N_e}E(i,j,k), \tag{1}$$

where

$$E(i,j,k) = \int_{e_k}\frac{x^{i+1}z^{j+1}}{x^2+z^2}dz - \int_{e_k}\frac{x^i z^{j+2}}{x^2+z^2}dx. \tag{2}$$

Calculation of the $E(i,j,k)$ function depends on whether the kth segment of the polygon is parallel to the z-axis. Then they try to get an analytical solution (closed form — equations 18 and 25). The case $q=0$ is missed in the derivation of equations 19–22 in their paper, and equation 19 for I_0 is incorrect. In the derivation of their equation 25, the case that $j=0$ was missed.

Therefore, their closed-form equations do not apply to any density-contrast model that has terms containing only x.

In the following discussion, I provide the corrected analytical solution (closed form) at the origin of the coordinate system for the gravity anomaly calculation for 2D mass bodies. The density contrast is a polynomial function in x and z. For convenience of comparison, all notations are the same as in their paper.

When the kth segment is not parallel to the z-axis, inserting their equations 11 and 12 in equation 2 and coalescing the two integrals into one, it yields

$$E(i,j,k) = -\sum_{l=0}^{j+1}C_{j+1}^{l}p^{j-l+1}q^{l+1}I_{i+j-l+1}, \tag{3}$$

where

$$I_n = \int_{x_k}^{x_{k+1}}\frac{x^n}{cx^2+bx+a}dx. \tag{4}$$

Here, $a=q^2$, $b=2pq$, and $c=1+p^2$.

For $Q=4ac-b^2=4q^2\geq 0$, two cases must be considered. The first case is $Q=0$, i.e., $q=0$. If $q=0$, equation 3 becomes

$$E(i,j,k) = 0, \tag{5}$$

which holds for any $i\in[0,N_x]$, $j\in[0,N_z]$, where N_x and N_z are given in their equation 1.

The case for $q=0$ is missing in the derivation of equations 19–22 in their paper. For instance, consider the contribution of the first term of the density-contrast model in their equation 1. For this case, $i=0, j=0$. From their equation 18 and for any k, I obtain $E(0,0,k)\neq 0$. This means that q is not a multiplying factor and $E(i,j,k)=0$ cannot be derived from their equation 18 for all combinations of valid values of i and j. Consequently, when $q=0$, the error caused by $E(0,0,k)\neq 0$ would propagate into the final results if their equation 18 were used.

The second case is $Q>0$. The integral in equation 4 is recursive because

$$\int\frac{x^n}{a+bx+cx^2}dx = \frac{x^{n-1}}{(n-1)c} - \frac{b}{c}\int\frac{x^{n-1}}{a+bx+cx^2}dx - \frac{a}{c}\int\frac{x^{n-2}}{a+bx+cx^2}dx$$

(Beyer, 1984). The series of the recursive integrals is given as

$$I_0 = \frac{1}{|q|}\left(\tan^{-1}\frac{(1+p^2)x_{k+1}+pq}{|q|} - \tan^{-1}\frac{(1+p^2)x_k+pq}{|q|}\right), \tag{6}$$

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$$I_1 = \frac{1}{(1+p^2)} \ln \frac{r_{k+1}}{r_k} - \frac{pq}{1+p^2} I_0, \qquad (7)$$

$$I_n = \frac{1}{(n-1)(1+p^2)}(x_{k+1}^{n-1} - x_k^{n-1}) - \frac{2pq}{1+p^2} I_{n-1} - \frac{q^2}{1+p^2} I_{n-2}, \quad (n > 1). \qquad (8)$$

The values I_1 and I_n are the same as equations 20 and 22 of Zhang et al. (2001), but I_0 given by equation 6 is the corrected equation for their equation 19:

$$E(i,0,k) = x_k^{i+1}(\ln z_{k+1} - \ln z_k) - \frac{x_k^{i-1}}{2}(z_{k+1}^2 - z_k^2) + x_k^{i+1} \ln\left(\frac{r_{k+1}}{r_k}\right),$$

where l'Hôpital's rule is used to find the right-hand side of the equation when $j \to 0$. From the following discussion (equations 9 and 11), when $j = 0$, $E(i,0,k)$ should be $x_k^{i+1} \ln(r_{k+1}/r_k)$. The consequence of missing the case $j = 0$ is that their equations 25–27 do not apply to any density-contrast model that includes any term containing only x.

When the kth segment is parallel to the z-axis, the $E(i,j,k)$ function is given by their equation 23:

$$E(i,j,k) = x_k^{i+1} K_{j+1}, \qquad (9)$$

where

$$K_n = \int_{z_k}^{z_{k+1}} \frac{z^n}{x_k^2 + z^2} dz. \qquad (10)$$

For $j = 0, n = 1$,

$$K_1 = \int_{z_k}^{z_{k+1}} \frac{z}{x_k^2 + z^2} dz = \ln\left(\frac{r_{k+1}}{r_k}\right). \qquad (11)$$

For $j = 1, n = 2$,

$$K_2 = \int_{z_k}^{z_{k+1}} \frac{z^2}{x_k^2 + z^2} dz = (z_{k+1} - z_k) - |x_k|\left(\tan^{-1}\left(\frac{z_{k+1}}{|x_k|}\right) - \tan^{-1}\left(\frac{z_k}{|x_k|}\right)\right). \qquad (12)$$

Using their equation 24 for $n > 2$, the recursive integral equation 10 becomes

$$K_n = \frac{1}{n-1}(z_{k+1}^{n-1} - z_k^{n-1}) - x_k^2 K_{n-2}, \quad (n > 2). \qquad (13)$$

Now, equations 1, 3, 5–9, and 11–13 form a complete set of analytical equations that can be used to calculate the gravity anomaly caused by a mass polygon at the origin of the coordinate system. The density contrast of the mass polygon satisfies the polynomial model $\sigma(x,z) = \sum_{i=0}^{N_x} \sum_{j=0}^{N_z} a_{ij} x^i z^j$, where constants a_{ij} are the coefficients of the polynomial.

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